Exercise 1.50

For the two vectors in Fig. E1.39, (a) find the magnitude and direction of the vector product $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$; (b) find the magnitude and direction of $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$.

Solution

The vectors in Fig. E1.39 were found in Exercise 1.39.

$$\mathbf{A} = \langle A_x, A_y \rangle = \langle 2.80 \cos 60^\circ, 2.80 \sin 60^\circ \rangle \text{ cm}$$
$$\mathbf{B} = \langle B_x, B_y \rangle = \langle 1.90 \cos 60^\circ, -1.90 \sin 60^\circ \rangle \text{ cm}$$

The vector (cross) product is obtained by evaluating a 3×3 determinant.

$$\begin{aligned}
\mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\
&= \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ 2.80 \cos 60^\circ \text{ cm} & 2.80 \sin 60^\circ \text{ cm} & 0 \\ 1.90 \cos 60^\circ \text{ cm} & -1.90 \sin 60^\circ \text{ cm} & 0 \end{vmatrix} \\
&= \begin{vmatrix} 2.80 \sin 60^\circ \text{ cm} & 0 \\ -1.90 \sin 60^\circ \text{ cm} & 0 \end{vmatrix} \hat{\boldsymbol{i}} - \begin{vmatrix} 2.80 \cos 60^\circ \text{ cm} & 0 \\ 1.90 \cos 60^\circ \text{ cm} & 0 \end{vmatrix} \hat{\boldsymbol{j}} + \begin{vmatrix} 2.80 \cos 60^\circ \text{ cm} & 2.80 \sin 60^\circ \text{ cm} \\ 1.90 \cos 60^\circ \text{ cm} & -1.90 \sin 60^\circ \text{ cm} \end{vmatrix} \hat{\boldsymbol{k}} \\
&= [(2.80 \sin 60^\circ \text{ cm})(0) - (0)(-1.90 \sin 60^\circ \text{ cm})]\hat{\boldsymbol{i}} - [(2.80 \cos 60^\circ \text{ cm})(0) - (0)(1.90 \cos 60^\circ \text{ cm})]\hat{\boldsymbol{j}} \\
&\qquad + [(2.80 \cos 60^\circ \text{ cm})(-1.90 \sin 60^\circ \text{ cm}) - (2.80 \sin 60^\circ \text{ cm})(1.90 \cos 60^\circ \text{ cm})]\hat{\boldsymbol{k}} \\
&\approx 0\hat{\boldsymbol{i}} - 0\hat{\boldsymbol{j}} + (-4.61 \text{ cm}^2)\hat{\boldsymbol{k}} \\
&\approx -4.61 \text{ cm}^2 \hat{\boldsymbol{k}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B} \times \mathbf{A} &= -(\mathbf{A} \times \mathbf{B}) \\
&= -(-4.61 \text{ cm}^2 \hat{\boldsymbol{k}}) \\
&= 4.61 \text{ cm}^2 \hat{\boldsymbol{k}}
\end{aligned}$$

The magnitudes of $\mathbf{A} \times \mathbf{B}$ and $\mathbf{B} \times \mathbf{A}$ are both about 4.61 cm^2, and they point in the negative and positive z-directions, respectively.